

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 2

Thursday 4 June 2020

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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8	
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10	
11	
12	
13	
14	
15	
TOTAL	



Answer **all** questions in the spaces provided.

- 1** Three of the four expressions below are equivalent to each other.

Which of the four expressions is **not** equivalent to any of the others?

Circle your answer.

[1 mark]

$$a \times (a + b)$$

$$(a + b) \times b$$

$$(a - b) \times b$$

$$a \times (a - b)$$

- 2** Given that $\arg(a + bi) = \varphi$, where a and b are positive real numbers and $0 < \varphi < \frac{\pi}{2}$, three of the following four statements are correct.

Which statement is **not** correct?

Tick (✓) **one** box.

[1 mark]

$$\arg(-a - bi) = \pi - \varphi$$

☐

$$\arg(a - bi) = -\varphi$$

☐

$$\arg(b + ai) = \frac{\pi}{2} - \varphi$$

☐

$$\arg(b - ai) = \varphi - \frac{\pi}{2}$$

☐


3 Find the gradient of the tangent to the curve

$$y = \sin^{-1} x$$

at the point where $x = \frac{1}{5}$

Circle your answer.

[1 mark]

$$\frac{5\sqrt{6}}{12}$$

$$\frac{2\sqrt{6}}{5}$$

$$\frac{4\sqrt{3}}{25}$$

$$\frac{25}{24}$$

Turn over for the next question

Turn over ►



The matrices **A** and **B** are defined as follows:

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

[3 marks]

[illegible]

Solve the inequality

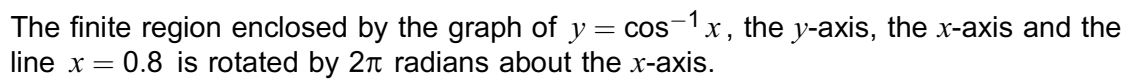
[5 marks]

[illegible]

Find the sum of all the integers from 1 to 999 inclusive that are not square or cube numbers.

[illegible]

The diagram shows part of the graph of $y = \cos^{-1} x$



[5 marks]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Factorise

[6 marks]

[illegible]

Jun20/7367/2

The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where a and b are positive real numbers,

$$\text{and } \mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Use **C** to show that $\cos \frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where m and n are integers.

[illegible]

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Turn over ►



The sequence $u_1, u_2, u_3, \dots$ is defined by

Prove by induction that, for all integers $n \geq 1$,

[6 marks]

[illegible]

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Turn over ►



$$\frac{\sin x}{x} - \cos x$$
$$(-1)^{r+1} \frac{2r}{(2r+1)!} x^{2r}$$

[4 marks]

[illegible]

11 (b) Show that

$$\lim_{x \rightarrow 0} \left[\frac{\frac{\sin x}{x} - \cos x}{1 - \cos x} \right] = \frac{2}{3}$$

[4 marks]

This image shows a single page from a notebook or ledger. The page is white and features approximately 20 evenly spaced horizontal blue lines running across its width. There are no vertical margin lines, text, or other markings present on the page.

Turn over ►



Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that

where q and r are rational numbers to be found.

[6 marks]

[illegible]

A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{dv}{dt} + v = 5e^t \sin t$$

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

[6 marks]

[illegible]

Charlotte is trying to solve this mathematical problem:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 10e^{-2x}$$

Particular integral: $y = \lambda e^{-2x}$

$$\frac{dy}{dx} = -2\lambda e^{-2x}$$
$$\frac{d^2y}{dx^2} = 4\lambda e^{-2x}$$

[2 marks]

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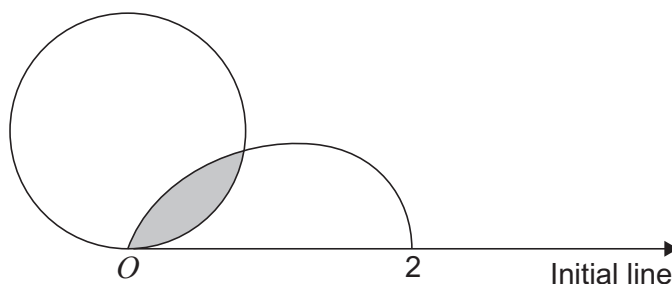
Explain how Charlotte should have started her solution differently and find the general solution of the differential equation.

[illegible]

14

The diagram shows the polar curve C_1 with equation $r = 2 \sin \theta$

The diagram also shows part of the polar curve C_2 with equation $r = 1 + \cos 2\theta$

**14 (a)**

On the diagram above, complete the sketch of C_2

[2 marks]

14 (b)

Show that the area of the region shaded in the diagram is equal to

$$k\pi + m\alpha - \sin 2\alpha + q \sin 4\alpha$$

where $\alpha = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$, and k , m and q are rational numbers.

[9 marks]



[illegible]

[3 marks]

[illegible]

[2 marks]

[4 marks]

15 (c) The line L_2 lies in the plane Π , and every point on L_2 is equidistant from A and B .

Find an equation of the line L_2

[4 marks]



[3 marks]

[illegible]

There are no questions printed on this page

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



Question number	
	Additional page, if required. Write the question numbers in the left-hand margin.
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